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VERTEX RECONSTRUCTION WITHOUT TRACK RECONSTRUCTION (STRAIGHT TRACKS)

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The determination of the vertex position without the previous reconstruction of all straight trajectories (the global maximum position of the "vertex function") is described. The method is based on the use of the discrete function of an accuracy and in the integration on the approximately-rectangular square. The analysis of the "Monte-Carlo" events is presented.

The investigation has been performed at the Scientifical-Methodical Division, JINR.

Определение вершин без восстановления траекторий (прямые треки)

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Представлен способ определения координат вершины без предварительного восстановления всех прямых траекторий (определение глобального максимума "вершинных функций"). Метод основан на использовании дискретных функций точности и на интегрировании по приближенно-прямоугольной площади. Приводится анализ "Монте-Карло" событий.

Работа выполнена в Общеинститутском научно-методическом отделении ОИЯИ.

In papers /1,2/ the "vertex functions" (VF) that permiy one to determine vertex coordinates without previous reconstruction of all trajectories were proposed. In particular implementation of VF for straight trajectories /2/, in case when the parameters of the primary single particle trajectory are known, can be considered successfull. Functions of such type — "vertex function of primary interaction" (FPI) — are multiexternal. The main (global) maximum position corresponds to the interaction point, and principal difficulty in implementation of VF is to localize the global maximum (GM) region. To resolve this task for FPI an "integral" method /3,4/ that requires analytic calculation of the integrals (moments) for the analyzing function $\phi(\vec{r})$ was used —

$$\mu_n(\phi) = \mu_o^{-1}(\phi) \cdot \int_V (\vec{r} - \vec{a})^n \phi(\vec{r})\, dv(\vec{r}), \quad \vec{a} \equiv \mu_o^{-1} \int_V \vec{r}\, \phi(\vec{r})\, dv, \tag{1}$$

$$n = 0, 1, 2, \ldots, 4; \qquad \mu_o(\phi) \equiv \int_V \phi(\vec{r})\, dv(\vec{r}),$$

where "V" is the vertex position region.

The analysis of GM of one-dimensional FPI — C(z) (z is the primary vertex coordinate) is given in /2/. This work is devoted to the GM position determination of two-dimensional VF in the detector systems that register straight trajectories:

$$D(x, z) = \sum_{k=1}^{M_N} \sum_{n=1}^{N-1} \sum_{m=1}^{M_n} \sigma_{mn} \cdot G[\frac{x - a_{kN}}{z_N - z} \cdot (z_N - z_n) + a_{kN} - a_{mn}; \sigma_{mn}]. \tag{2}$$

Each n-th detector ($n = 1,2,\ldots,N$) placed in z_n-position on the z-axis registers M_n of X-coordinates a_{mn}($m = 1,2,\ldots, M_n$) with the accuracy (σ) function $G(t;\sigma)$. The function $G(t;\sigma)$ can look like: discrete —

$$G(t;\sigma) = \begin{cases} (2\sigma)^{-1}, & |t| \leq \sigma \\ 0, & |t| > \sigma, \end{cases} \tag{3a}$$

smooth —

$$G(t;\sigma) = (2\pi\sigma^2)^{-1/2} \cdot \exp(-t^2/2\sigma^2), \tag{3b}$$

or partially-smooth —

$$G(t;\sigma) = \begin{cases} \frac{3}{\sigma 4\sqrt{5}} (1 - t^2/5\sigma^2), & |t| \leq \sigma\sqrt{5} \\ 0, & |t| > \sigma\sqrt{5}. \end{cases} \tag{3c}$$

The GM position ($x = u$, $z = v$) of the function (2) corresponds to the vertex coordinates of primary (for example) interaction. It should be noted that vertex (u, v) determined by (2) may serve as an additional detector (with single count) for posterior reconstruction of trajectories. This fact initiates the attempts to utilise the VF possibilities.

Determination of the Integral Momenta

The integrals (1) calculation is one of the complicated "technological" stage in this task. To determine momenta (1) it is necessary to fulfill analytic integration:

$$I_{\mu\nu}\ (k, m, n) = \iint_S dx\,dz(x-p)^{\mu}\ (z-q)^{\nu}\ G\left[\frac{x-a_{kN}}{z_N - z}(z_k - z_n) + a_{kN} - a_{mn};\sigma_{mn}\right], \qquad (4)$$

$$\mu, \nu = 0,1,2, \ldots, 4; \quad \nu + \mu \leq 4$$

$S\ (S_L \leq x \leq X_U;\ Z_L \leq z \leq Z_U)$ is the vertex search region, p, q are coordinates of the center of $D(x, z)$. This task becomes simpler if we use discrete (3a) function of accuracy $G(t;\sigma)$ and allow some distortion of rectangle S (S increases).

This approximations can be illustrated for the particular case, when the detector (z_k) is located on the left of the S-region (i.e. $z_k < Z_L$) and, besides, $X_L > 0$. The S' integration area in variables: $t = z - z_k$, $y = (x - a_{kN})/t$ is given in fig. 1. The S'-area boundaries are defined by the points: $P_1 = (X_U - a_{kN})/(Z_L - z_k)$, $P_2 = (X_U - a_{kN})/(Z_U - z_k)$, $P_3 = (X_L - a_{kN})/(Z_L - z_k)$, $P_4 = (X_L - a_k)/(Z_U - z_k)$. The integral (4) has a non-zero value if the strip $A_1 \leq y \leq A_2$ intersects the curvilinear S'-area, the strip boundaries are determined by the accuracy function parameters:

$$A_1 \equiv T + R, \quad A_2 \equiv T - R; \quad T \equiv \frac{a_{mn} - a_{kN}}{z_z - z_k}, \quad R \equiv -\frac{\sigma_{mn}}{|z_n - z_k|}. \qquad (5)$$

The primary integration area distortion is nonsignificant if the parameter R(5) is of small value, i.e. if the accuracy (σ) of the detectors is high. In such an approximation, the maximal common surface of the strip (A_1, A_2) and S' (dashed rectangular in fig. 1) is easily determined and the integration result (4) has a simple form:

$$I_{\mu\nu} = \frac{1}{\mu+1} \sum_{k=0}^{\mu+1} \frac{C^k_{\mu+1}}{k+\nu+1}\, t^{k+\nu+1} \Big|_c^d\ A^k (a_k - p - qA)^{\mu+1-k} \Big|_{A_1}^{A_2} \qquad (6)$$

(C^k_N — binominal coefficients; c, d — the rectangular boundaries).

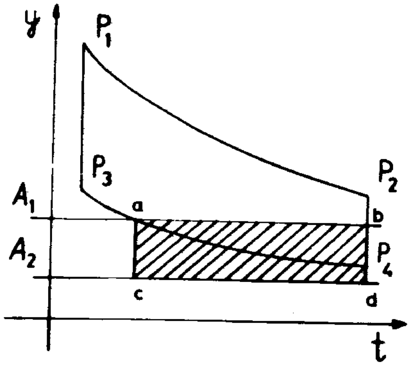


Fig. 1. The integration region in the curvilinear coordinates. The dashed rectangular is the region for the calculation of the integral momenta of the vertex function for a measured point, that can belong to a trajectory emmitting from the vertex.

Iterational Vertex Coordinates Determination

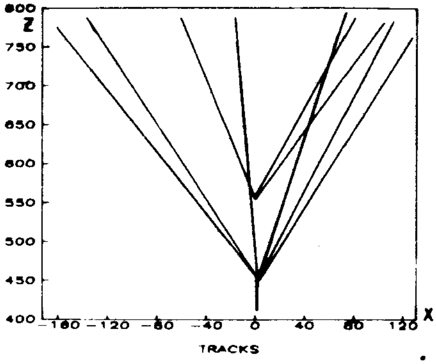
The determining momenta (1) are used further to analyse the excess [4, p.50], that is the function of the vector indicating the subregion of GM of VF (2). As it has been noted in [4], the creation of the universal iteration procedure for the reliable localization of the GM-subregion is an open question. However, if some specific direction in a track experiment exists (for instance, the direction of the primary particle in one-beam accelerators), then one can propose the receipt to choose one of the 6 possible vectors, corresponding to the external excess values — the vector with minimal value of z -component should be chose. This choice is proved to be correct in the primary vertex determination, because vertices with higher values of z -component are secondary ones.

The chosen vector of the approximate vertex position is used as initial point for the well-known gradient method (for instance) for the exact determination of the vertex position. In the gradient method the partially-smooth function of the accuracy (3c) can be used to avoid the well-known properties in the behaviour of the second derivations of smooth function (3b). The criterion of the iteration termination is not only the given deviation value between two consequent iterations, but also the amplitude of $D(x, z)$ at the GM-position if the estimation of number of tracks of the event is known ($D_{max} \simeq (N - 1) \cdot M$, M is the number of detectors, M is the number of tracks).

The Image Defocusing

To search for the vertex "globally" and to precise it "locally", the method of function (2) "defocusing" by the change of the accuracy parameter (σ) can be used. For the first iteration this parameter (σ) is in-

Fig. 2. "Monte-Carlo" event with 2 vertices. All tracks are ideal straight lines.



tentially "decreased" — $\sigma' = \sigma \cdot f$, $f > 1$ ($f = 3$ or 2) is the increase factor ("defocusing"). This method results to a more "soft" image of the function, it means that the "high-frequency" component of D(x, z) is suppressed. This "high-frequency" component may be somewhat "dangerous" for both "global" and "local" vertex search. As to that, the next example is very significant: A track event with 2 vertices is shown in fig. 2 ($z_1 = 450$ mm, $x_1 = 0$ mm; $z_2 = 550$, $x_2 = -5$ mm), 3 detectors with z = 0, 200, 400 mm register the coordinates of a single track of primary particle (these coordinates are also included in VF); the detectors (z = 600, 800, 1000, 1200 mm) register the coordinates of 9 secondary particles with the conditional accuracy $\sigma = 1$ mm (all tracks are ideal straight lines). The function D(x, z) for defocusing factor f = 2 is shown in fig. 3a and for f = 0.5 — in fig. 3b. Indeed, the main maximum region in fig. 3a is practically unimodal to compare to analogous region of fig. 3b. In following iterations the defocusing factor can be decreased up to some limit (for instance, $f_{min} = 0.5$).

It should be noted that the parameter σ reflects not only the detector accuracy, but also the degree of resolution of the whole track pattern, that proves such a "carefull" deal with vertex search.

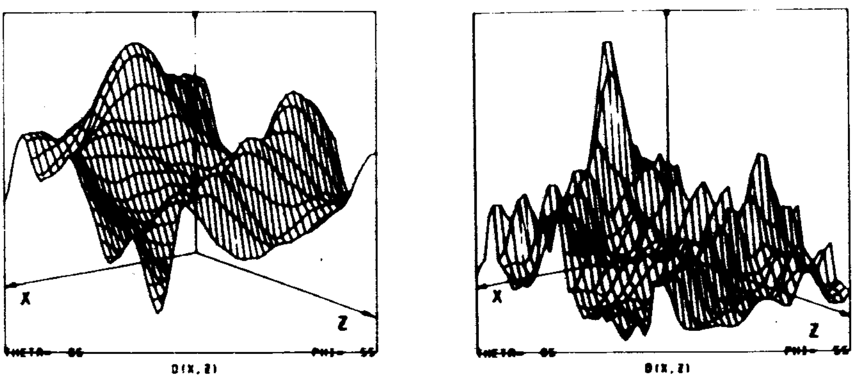


Fig. 3a,b. The vertex function of the event shown in fig. 2 at the various values of "defocusing" factor: f = 2 (fig. 3a) and f = 0.5 (fig. 3b).

Vertex Determination Efficiency

To verify a method of the primary vertex determination 100 simplest Monte-Carlo events where analyzed, as it is shown in fig. 2. The secondary vertex played a role of the noise source. The criterium of vertex coordinated (u, v) determination precision was the statistic distribution of the distance between the found vertex and given ones (x_0, z_0)

$$R = \sqrt{(u - x_0)^2 + (v - z_0)^2}.$$

The typical distribution of the parameter (R) is given in fig. 4, the number of detectors registering secondary particles is $N_1 = 10$ and $f = 3$. The relative number (E %) (to all analyzed events), when $R < R_1$, has the behaviour: E(R < 1 mm) = 91%, E(R < 5 mm) = 99%. If multiplicity is fixed, then the precision depends on the number of detectors, — for $N_1 = 4$ the efficiency decreases: E(R < 1 mm) = 72 %, E(R < 5 mm) = = 86 %.

When there is no noise (secondary vertices), the efficiency is high enough: E(R <1 mm) < 99 %, E(R < 5 mm) = 100 %.

The analysis has been performed on 4.77 MHz IBM PC/XT-compatible computer. 100 events like those shown in fig. 2 were analyzed: 1 track of primary particle (3 detectors), 9 tracks of secondary particles (10 detectors). It took about 24 seconds per event. It should be noted, that it took about 80 % of CPU-time to calculate integrals (6), which is logically simple, whereas the most logically complicated part of the task — the determination of the integration limits — required only about 17 % of CPU-time. The calculation of (6) in the programm D(x, z) is not optimal at present: in the assumption $\sigma = 0$ the expression of (6) is more simple and CPU-time is 18 sec per event (however, the efficiency decreases: E(R < 5 mm) = 96 %).

Undoubtedly, this programm of D(x, z) -analysis should be improved substantially.

Nevetherless, one can hope that this method of the vertex determimation may be usefull for high multiplicity (M ~ 100) and in the noise condition.

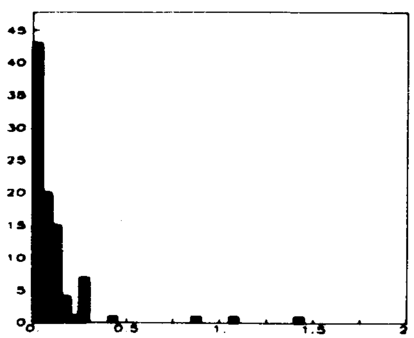


Fig. 4. The statistic distribution of the distance between the found vertex position and predetermined for the 100 "Monte-Carlo" like shown in fig. 2.

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References

1. Yatsunenko Yu.A. — JINR, P1-88-29, Dubna, 1988.
2. Yatsunenko Yu.A. — JINR, E1-88-907, Dubna, 1988.
3. Yatsunenko Yu.A. — JINR, P5-88-74, Dubna, 1988.
4. Yatsunenko Yu.A. — In: JINR Rapid Communications, No.7(33)-88, Dubna, 1988, p.47.

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